Distributed Search on Graphs Using Discrete Time Quantum Walk

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1. Introduction

Searching with coined quantum walk is a problem that has interested the community since a long time. While most results consider spatial searches on regular lattices, some work have introduced several models of coined quantum walks on graphs. This work introduces a distributed searching quantum walk on graphs. Our contribution is in two parts: (i) we introduce a new mathematical model of a coined quantum walk on graphs designed to search both nodes or edges; (ii) we provide an anonymous distributed scheme to implement such a model.

2. Quantum Walk on Graphs

We consider an undirected graph $G = (V, E)$ and define a walker on its edges. The coin is defined as a 2×2 unitary operator C on the coin register :

$$\forall (u,v) \in E,\ |(u,v)\rangle\,|\pm\rangle \overset{\text{coin}}{\longmapsto} (I \otimes X) \times |(u,v)\rangle\,|\pm\rangle\,.$$

The scattering operator coincides instead with the standard diffusion operator D_n of size $n \times n$ applied locally on each node :

$$\forall u \in V,\ \left(\psi_{u,v}^{\sigma(u,v)}\right)_{v\in V} \overset{\text{scattering}}{\longmapsto} D_{\deg(u)} \times \left(\psi_{u,v}^{\sigma(u,v)}\right)_{v\in V},\ \text{where } D_n = \left(\frac{2}{n}\right)_{i,j} - I_n.$$

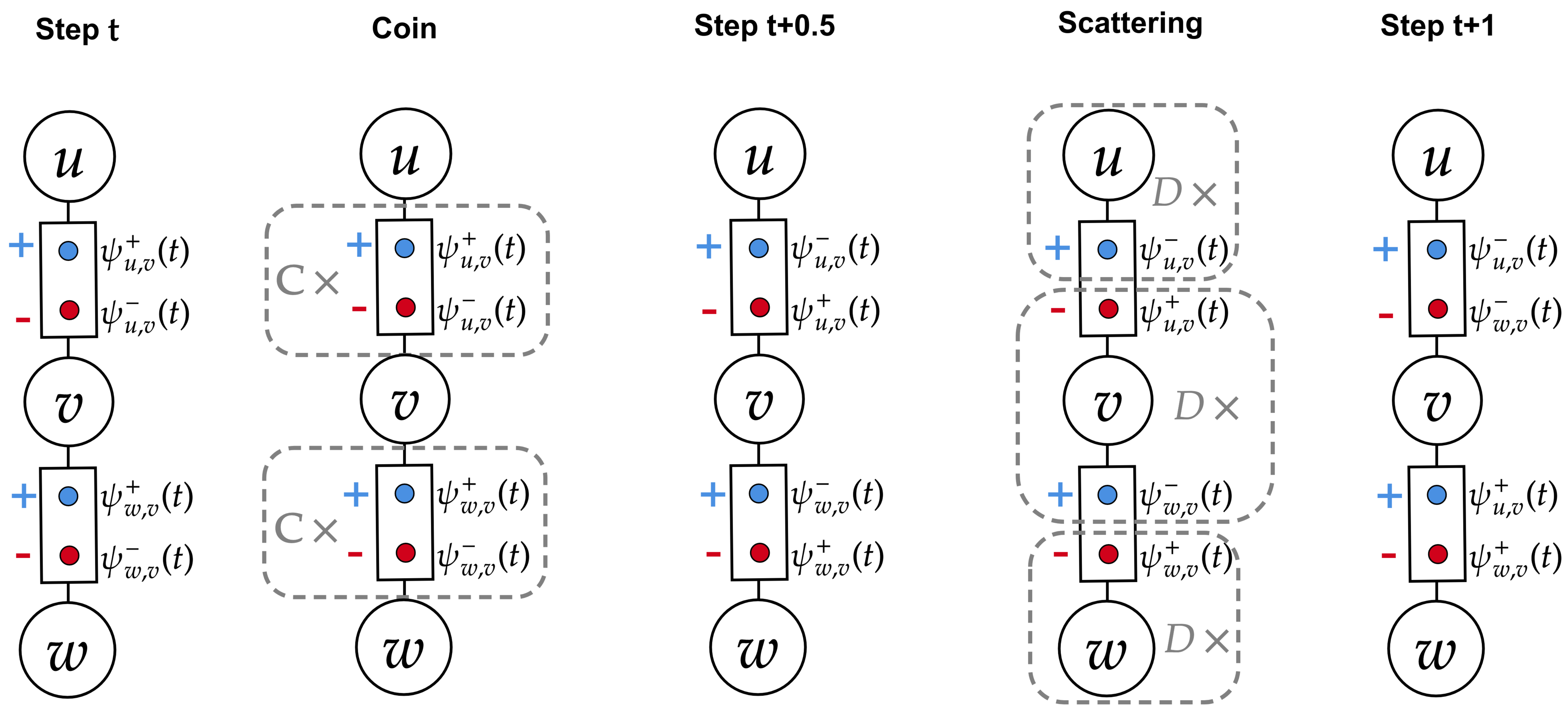


Figure 1: Example of a walk on a path of size 3.

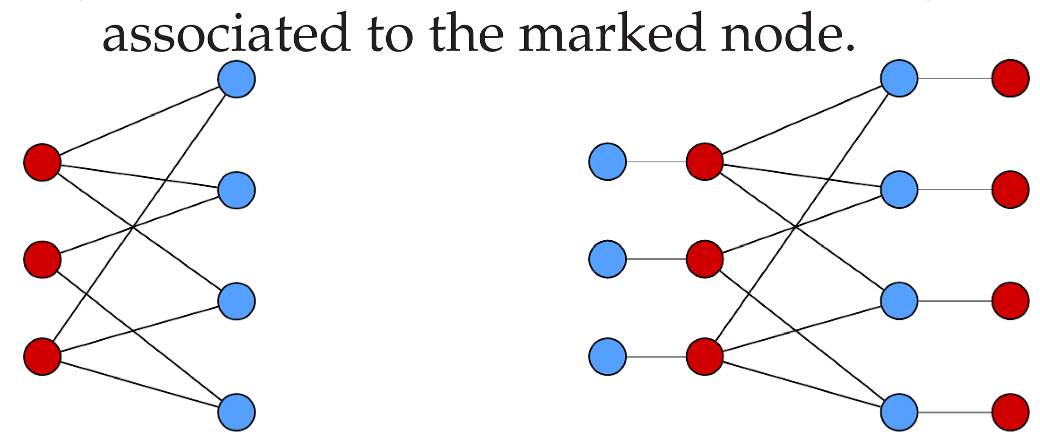
3. Searching an Edge

We add an oracle operation, which applies X to the marked edge and I everywhere else. The Quantum Walk runs for T steps before a position measurement returns an edge. If the edge returned isn't the marked one, we repeat the process again.

The average runing time to search one edge in a graph is $O(T/P)$, where T is the hitting time of the walk and P the probability of measuring the marked edge after T steps.

4. Searching a Node

To search nodes instead of edges, we starify the graph and search for the virtual edge associated to the marked node.



(a) Initial Graph G. **(b)** Starified graph $\tilde{G}$.

Figure 2: Transformation from the initial graph to the starified one. The starified graph is the initial graph to which we added one virtual node per real node. Every virtual node is connected to (and only to) the associated real node with a virtual edge.

5. Complexities

2D-Grid	Hypercube	Complete Graph	Scale-Free Graph
$O\left(\sqrt{M \log^\star M}\right)$	$O\left(\sqrt{M}\right)$	$O\left(\sqrt{M}\right)$	$O\left(\sqrt{M}\right)$
$O\left(\sqrt{N \log^\star N}\right)$	$O\left(\sqrt{N \ln N}\right)$	$O(N)$	$O\left(\sqrt{N+M}\right)$

Table 1: Complexity of the QWSearch for several graphs in function of M. Results are analytical for the complete graphs and numerical for the others. The classical complexity of a BFS algorithm is $O(M)$

6. Distributed Scheme

The distributed implementation uses an edge register of two qubits per edge, one for the $+$ state and one for the $-$ state. This register starts in the W state (i.e. $(|001\rangle + |010\rangle + |100\rangle)/\sqrt{3}$).

We also need a node register, composed of $1 + \log d$ qubits per node of degree d, to serve as ansclary qubits during the scattering.

At the end of the computation, the edge register is measured.

7. Circuit

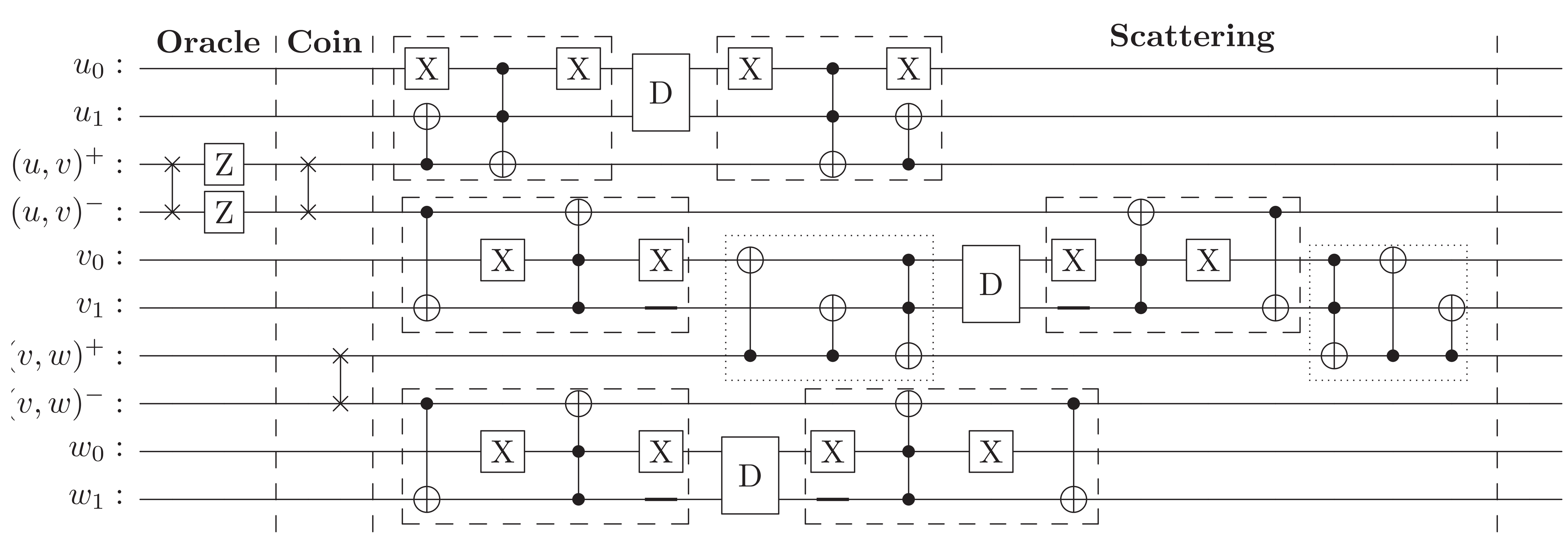


Figure 3: Circuit of one step of the quantum walk for the path graph $u - v - w$. Dashed lines signal Tr_1 circuit and its inverse while dotted lines Tr_2. The circuit applies successively the oracle on (u,v), the coin, Tr, D, Tr^{-1}.

8. Conclusion

This work introduces a new model of quantum walk on graph. This model is well suited to solve searching problems targetting edges or nodes. Additionally, a distributed implementation is provided.

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