# A METHOD FOR MONITORING THE AVERAGE POSITION OF A NEIGHBORHOOD HISTORY QUANTUM WALK

{ MATHIEU ROGET, GIUSEPPE DI MOLFETTA }  ENS DE LYON, AIX-MARSEILLE UNIVERSITY

## INTRODUCTION

A **Neighborhood History Quantum Walk (NHQW)** is a discrete time quantum walk (QW) in one dimension but with one additional memory qubit for each site of the walk. This model was introduced by Asif Shakeel[1]. Let $\mathcal{L} = \{-n, -n+1, \ldots, n-1, n\}$ be the lattice of the walk and $\mathcal{V} = \{-, +\}$ be the two possible velocities (left and right). Then a state $\psi$ of the walk is a linear combination of

$$|x, v, m_{-n}, \ldots, m_n\rangle\,,\ \forall x \in \mathcal{L},\ v \in \mathcal{V},\ m_{-n}, \ldots, m_n \in \{0,1\}^{2n+1}\,.$$

We deduce that $\psi \in \mathcal{H}_{\mathcal{L}} \otimes \mathcal{H}_{\mathcal{V}} \otimes \mathcal{H}_M$ with $\mathcal{H}_{\mathcal{L}} = \mathbb{C}^{2n+1}$, $\mathcal{H}_{\mathcal{V}} = \mathbb{C}^2$ and $\mathcal{H}_M = \bigotimes_{k=-n}^{n} \mathbb{C}^2$ .
The transition operation is in two parts :

1. **Scattering ($S$)** : Sets the velocity and the memories of the two neighbors.
2. **Advection ($A$)** : Sets the position.

We can choose how memory will impact the velocity in the Scattering operator (more details on this model here [1]). We can also choose the initial state $\psi(0)$ of the walk and especially the initial memory. Our purpose is to encode the average position of the walker into the initial memory.

## FINDING THE AVERAGE POSITION

We take the initial state

$$\psi(0) = |0\rangle \otimes |-\rangle \otimes \left( \bigotimes_{x=-n}^{-1} A_{-x}\,|0\rangle + B_{-x}\,|1\rangle \right) \otimes \left( \bigotimes_{x=0}^{n} |0\rangle \right)$$

and a well-chosen scattering operator.
Then, we have the following formula for the average position of the walker :

$$\overline{x}(t) = -t \prod_{k=1}^{t} A_k^2 + \sum_{k=0}^{t-1} (-2k+t)\, B_{k+1}^2 \prod_{j=1}^{k} A_j^2 \qquad (1)$$

And if we suppose that $A_k$ and $B_k$ are real then $A_k = \sqrt{1 - B_k^2}$ and

$$\overline{x}(t) = -t \prod_{k=1}^{t} (1 - B_k^2) + \sum_{k=0}^{t-1} (-2k+t)\, B_{k+1}^2 \prod_{j=1}^{k} (1 - B_j^2) \qquad (2)$$

Now we want to find $A_k$ and $B_k$ such that $\overline{x}(t)$ corresponds to a given function.

## LINEAR AVERAGE POSITION

If we use equation (2) with $\forall k < -1,\ B_k = 0$. Then the average position is $x(t) = vt$ with $v = \left(1 - 2B_1^2\right)$ .
Figures 1 and 2 present some examples of position distribution (in blue) and the corresponding average position (in red) :

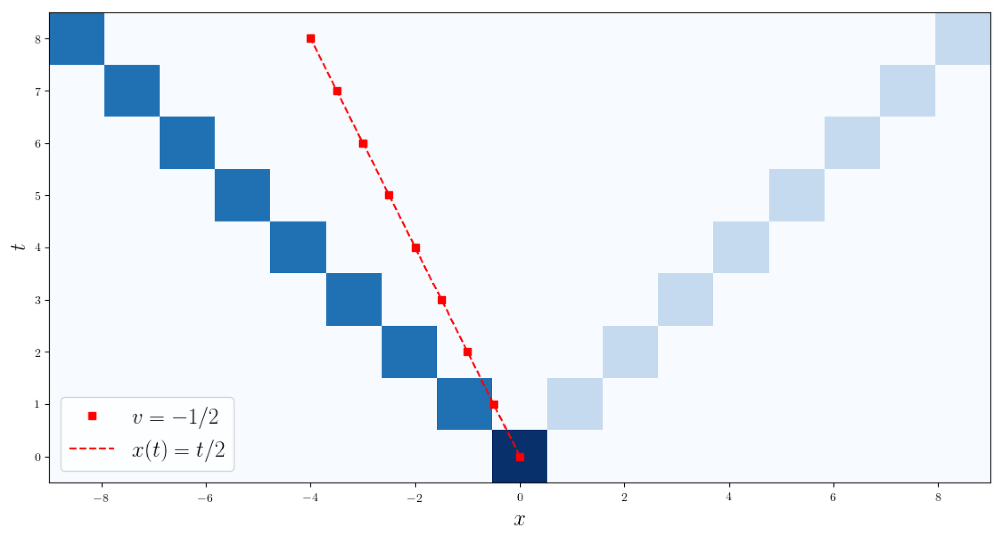

**Figure 1:** Distribution of linear trajectories

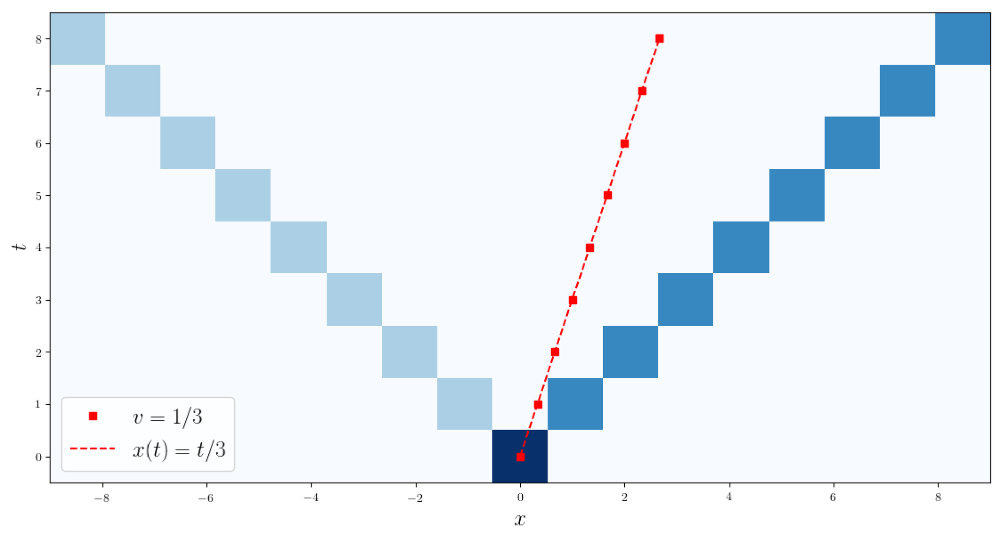

**Figure 2:** Distribution of linear trajectories

## QUADRATIC AVERAGE POSITION

If we use equation (2) with $B_k = \sqrt{-\dfrac{z}{z\,(k-1) - 2}}$ for some real $z$, then the average position is $\overline{x}(t) = -\dfrac{t\,(tz-2)}{z+2}$ . Figure 3 and 4 present some examples of position distribution (in blue) and the corresponding average position (in red) :

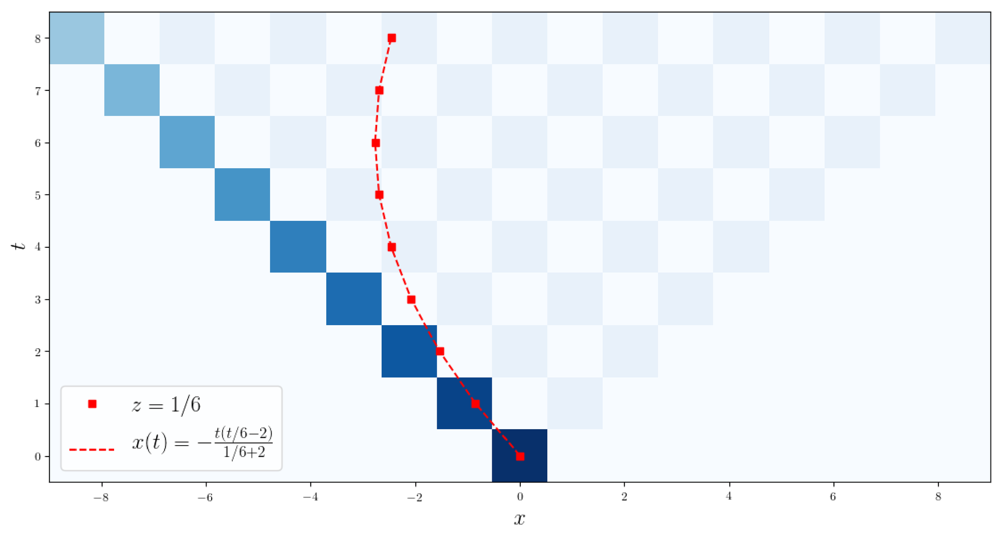

**Figure 3:** Distribution of linear trajectories

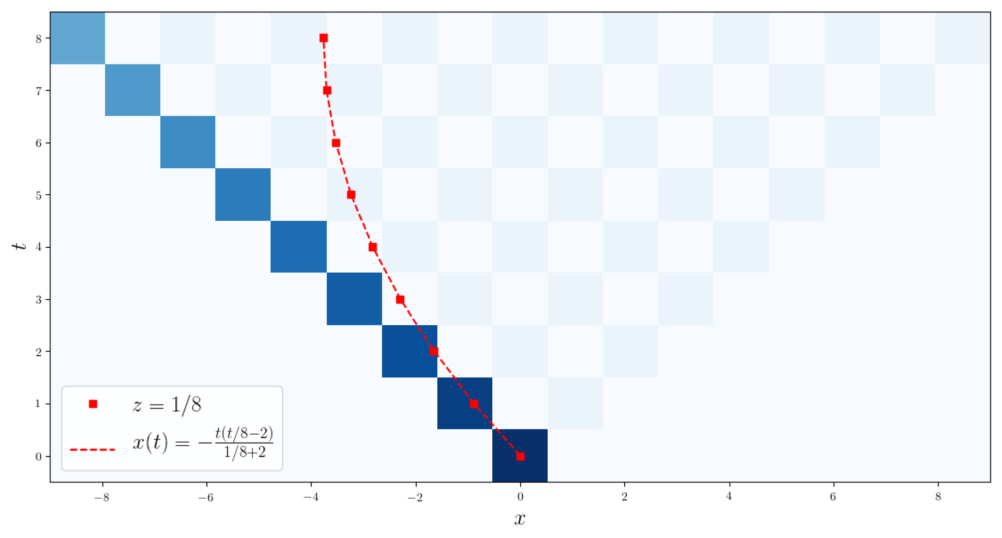

**Figure 4:** Distribution of linear trajectories

## CONCLUSION

In conclusion, we have a way of encoding the average position for quadratic or linear functions into the initial memory of the NHQW model. The important fact is that we don't need to supervise the walk or set their transition operator. We can set the average position just by fixing the initial memory. This result can be used to make some simulations with quantum walks. Problems become apparent when the number of steps exceeds $n$. In this case, because the walk is cyclic, the average position will not fit with the expected function.

## REFERENCES

[1] Asif Shakeel. Neighborhood-history quantum walk. *arXiv preprint arXiv:1611.07495*, 2016.

## CONTACT INFORMATION

**Email** mathieu.roget@ens-lyon.org
**Phone** 06 99 32 05 67