Quantum Perceptron Revisited: Computational-Statistical Tradeoffs

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1. Introduction

The perceptron is a popular machine learning algorithm that solves binary classification problems. It's complexity depends of how close the classes are (statistical complexity) and of the number of points (computational complexity). Wiebe et al [1] proposed two models of quantum perceptron that improve respectively the statistical or the computational complexity. We proposed a new model that improves both and discuss about the computational statistical tradeoff.

2. Grover algorithm

The Grover algorithm is a quantum algorithm that finds a marked element in an unstructured database of N elements in $O(\sqrt{N})$ steps. The version of Grover we use is one that allow the search of several elements.

- Probability to find a marked element : $\geq 1-\epsilon$
- Complexity : $O(\sqrt{N}\ln(1/\epsilon))$

3. Binary classification

The two hyperplanes on Figure 1 both separates the classes. The green one is more balanced and we call the distance between the most balanced separating hyperplane and the closest point the *margin* γ.

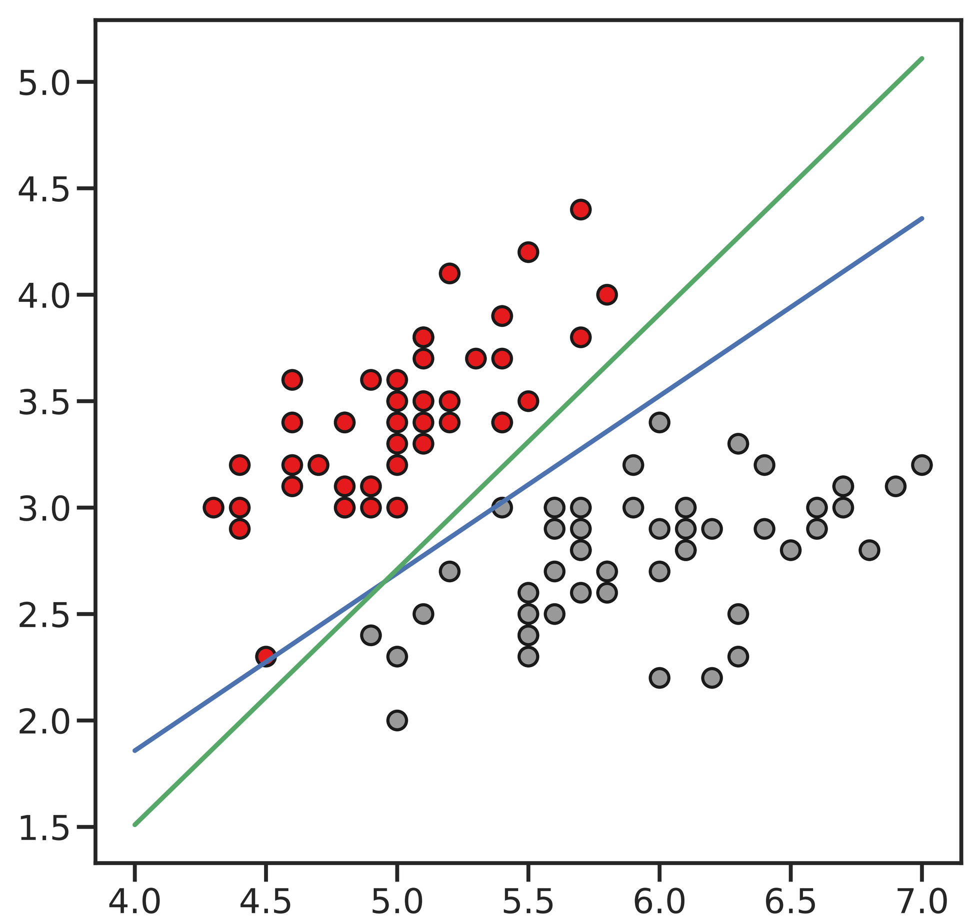

Figure 1: Example of binary classification on Iris dataset.

4. The algorithms

- Classical online perceptron: classical well known version of the perceptron. We test each point and update the hyperplane when an error is found.
- Online quantum perceptron: algorithm of Wiebe et al [1] that improves computational complexity. Same as the classical one but the search of a misclassified point is done via Grover's search.
- Version space quantum perceptron: algorithm of Wiebe et al [1] that improves statistical complexity. We draw several hyperplanes randomly and search via Grover's search for one that separates the classes.
- **Hybrid quantum perceptron: our algorithm that improves both complexities. We draw several hyperplanes randomly and for each, we check if it separates the classes.**

5. Complexity and generalization

Algorithm	Complexity	Expected risk
Classical online perceptron	$O\left(\frac{N}{\gamma^2}\right)$	$\leq \underset{S\sim\mathcal{D}^{N+1}}{\mathbb{E}}\left(\frac{\min(M(S),\frac{1}{\gamma_S^2})}{N+1}\right)$
Online quantum perceptron	$O\left(\frac{\sqrt{N}}{\gamma^2}\ln\left(\frac{1}{\epsilon\gamma^2}\right)\right)$	n/a
Version space quantum perceptron	$O\left(\frac{N}{\sqrt{\gamma}}\ln^{3/2}1/\epsilon\right)$	n/a
Hybrid quantum perceptron	$O\left(\frac{\sqrt{N}}{\gamma}\ln(1/\epsilon)\ln\left(\frac{1}{\gamma\epsilon}\right)\right)$	$\leq \underset{S\sim\mathcal{D}^{N+1}}{\mathbb{E}}\left(\sqrt{\frac{\pi}{2}}\frac{\ln 1/\epsilon}{N+1}\frac{1}{\gamma_S}\right)$

Table 1: Complexity and expected risk for each algorithm. The margin is noted γ, the number of points N and the probability of success $1-\epsilon$. The expected risk is defined by $\underset{S\sim\mathcal{D}^N}{E} R(h_S) = \underset{S\sim\mathcal{D}^N}{E}\underset{(x,y)\sim\mathcal{D}}{\mathbb{E}}(\mathbb{1}\{h_S(x)\neq y\})$

6. Simulations

Figure 2 shows ratio between the number of steps of the quantum algorithms and the number of steps of the perceptron. Two dataset have been used, the Iris dataset is a dataset easy to learn for the perceptron while the Hard dataset is specifiacally built to force the perceptron making a large number of steps.

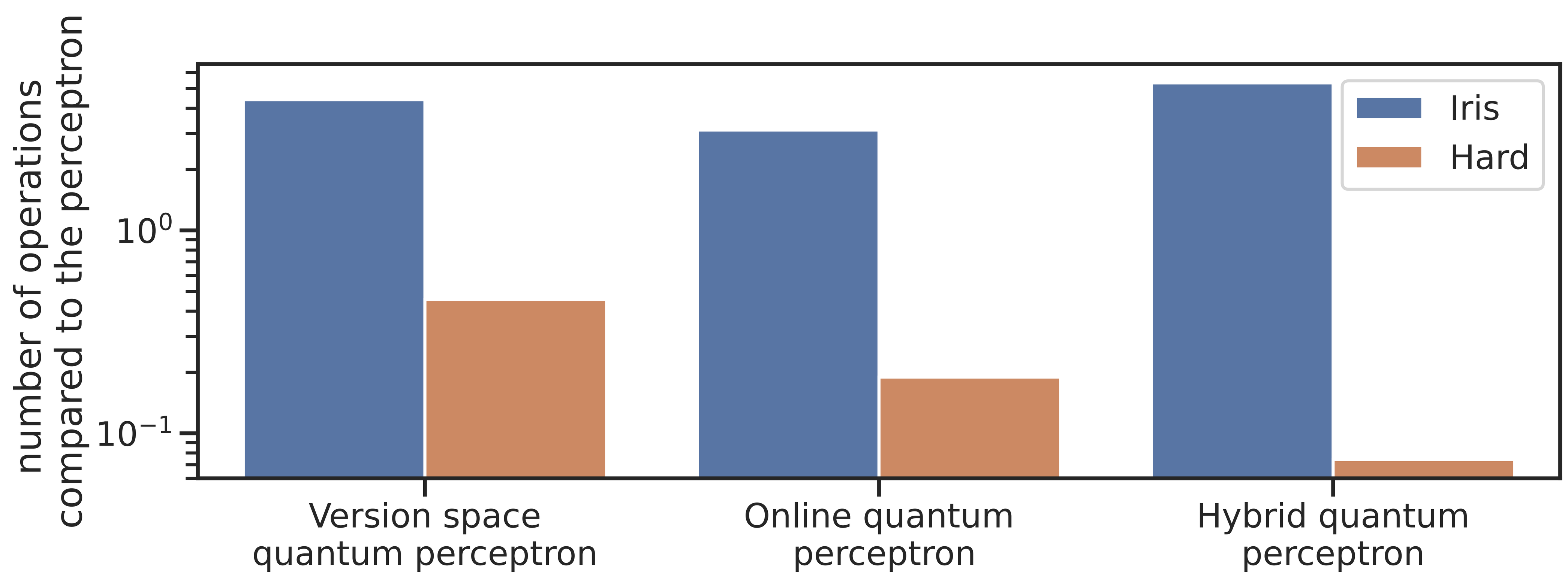

Figure 2: Ratio between the number of operations of quantum perceptron and classical perceptron.

7. Conclusion

In conclusion, the algorithm we propose provides *better performances* than it's classical counterpart both in term of *computational and statistical complexity*. The *generalization abilities also compares favorably* and simulations show better results on complex datasets. Further work can be made on the quantum implementation (quantum embedding) and the quantum noise.

References

[1] Nathan Wiebe, Ashish Kapoor, and Krysta M Svore. Quantum perceptron models. In *NeurIPS*, 2016.

[2] Mathieu Roget, Giuseppe Di Molfetta, and Hachem Kadri. Quantum perceptron revisited: Computational-statistical tradeoffs. In *UAI*, 2022.

[3] Mathieu Roget, Giuseppe Di Molfetta, and Hachem Kadri. Quantum perceptron revisited: Computational-statistical tradeoffs (supplementary materials). https://github.com/mroget/Quantum-perceptron-models, 2022.