Quantum Perceptron Revisited: Computational-Statistical Tradeoffs

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Motivation

- Quantum speedup refers to improving the complexity of a classical algorithm using quantum computation.

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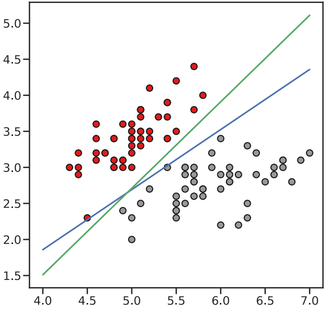
- The Grover's algorithm is a quantum algorithm used to search one element in an unsorted database of size N with complexity $O(\sqrt{N})$.
- In the case of the perceptron algorithm used for binary classification, we can wonder how the performances (computational/statistical complexities, generalization) are affected.

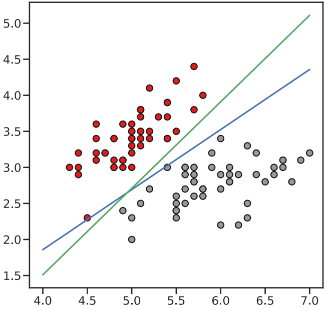
Binary linear classification



Binary linear classification

Sample :

$$S_N = \left\{(x_i, y_i) \mid 1 \leq i \leq N\right\} \sim \mathscr{D}^N$$



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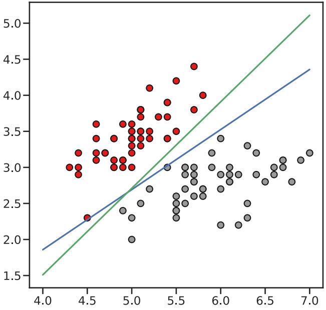

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Margin :

$$\gamma_S = \max_{v \in \mathbb{R}^D} \min_{(x,y) \in S} \frac{y\langle v, x\rangle}{\|v\|}$$

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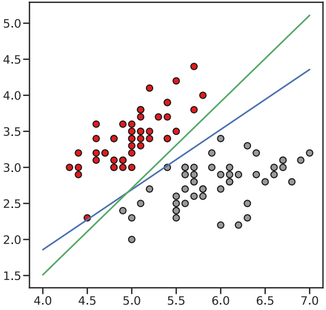

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Risk :

$$R(h_S) = \mathop{\mathbb{E}}_{(x,y)\sim\mathcal{D}} \left(\mathbb{1}\{h_S(x) \neq y\}\right)$$

Grover's algorithm for several marked elements

Aim : Search marked elements in unstructured database of size N.

Complexity : $O\left(\sqrt{N}\right)$.

Probability of success : $\geq \frac{1}{4}$

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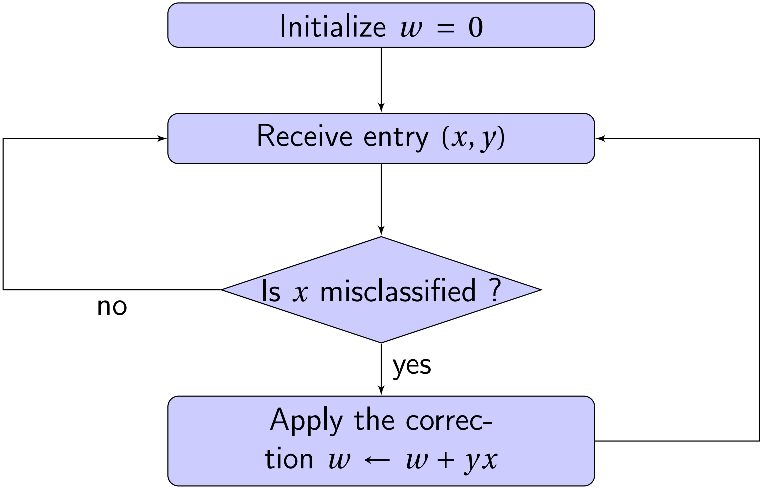
Increasing probability of success

One can arbitrary increase the probability of success by repeating the procedure a logarithmic number of times.

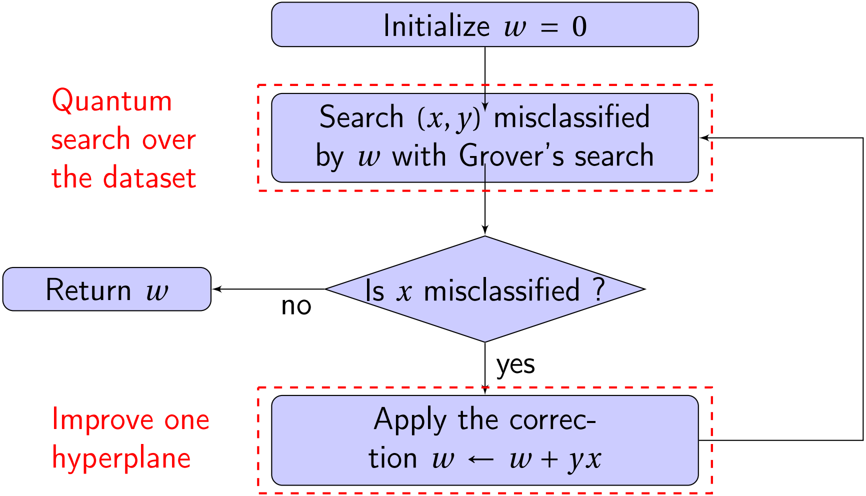
Classical Online Perceptron



Classical Online Perceptron

Algorithm	Complexity	Expected risk
Classical online perceptron	$O\left(\frac{N}{\gamma^2}\right)$	$\leq \mathop{\mathbb{E}}_{S\sim\mathscr{D}^{N+1}}\left(\frac{\min(M(S), \frac{1}{\gamma_S^2})}{N+1}\right)$
Online quantum perceptron		
Version space quantum perceptron		
Hybrid quantum perceptron (this work)		

Online Quantum Perceptron (Wiebe et al)



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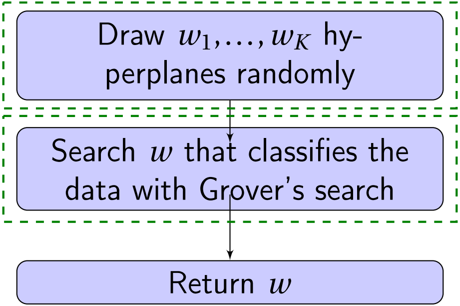
Version Space Quantum Perceptron (Wiebe et al)

Draw the hyperplanes instead of updating one

Quantum search over the hyperplanes



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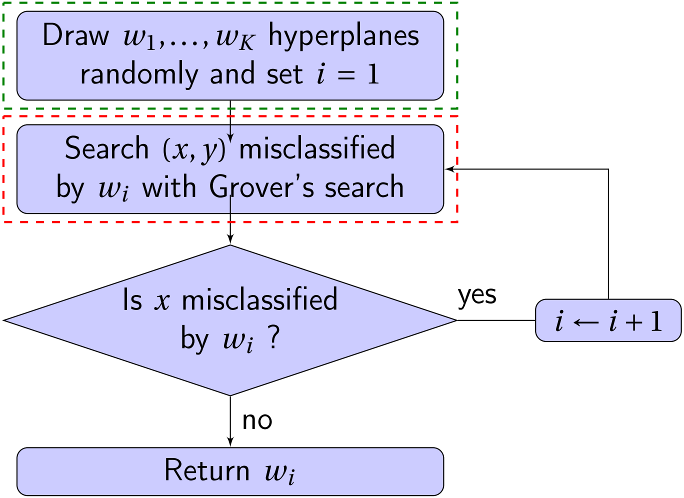
Quantum search over the dataset



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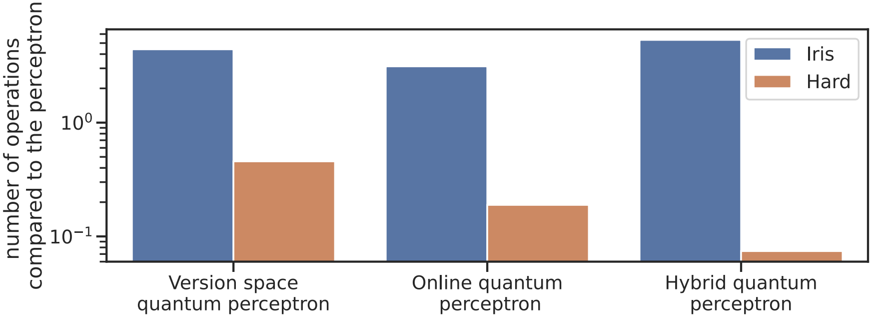
Version space quantum perceptron	$O\left(\frac{N}{\sqrt{\gamma}}\ln^{3/2} 1/\epsilon\right)$	n/a
Hybrid quantum perceptron (this work)	$O\left(\frac{\sqrt{N}}{\gamma}\ln\left(\frac{1}{\epsilon}\right)\ln\left(\frac{1}{\gamma\epsilon}\right)\right)$	$\leq \mathbb{E}_{S\sim\mathscr{D}^{N+1}}\left(\sqrt{\frac{\pi}{2}}\frac{\ln 1/\epsilon}{N+1}\frac{1}{\gamma_S}\right)$

Numerical experiments



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References

- Mathieu Roget, Giuseppe Di Molfetta, and Hachem Kadri. "Quantum Perceptron Revisited: Computational-Statistical Tradeoffs." In: *UAI*. 2022
- Nathan Wiebe, Ashish Kapoor, and Krysta M Svore. "Quantum perceptron models." In: *NeurIPS*. 2016